# Evaluation of 2-norm versus Sparsity Regularization in Spline-Based Joint Reconstruction of Epicardial and Endocardial Potentials from Body-Surface Measurements.

Jaume Coll-Font[1], Brittany Purcell[2], Jingjia Xu[2], Petr Stovicek[3], Dana H Brooks[1], Linwei Wang[2]

[1] B-spiral group in the ECE dept. at Northeastern University, Boston (MA), USA
[2] Rochester Insitute of Technology, Rochester (NY), USA
[3] Charles University Hospital, Prague, Czech Republic

## Abstract

*Cardiac electrical imaging, reconstruction of cardiac electrical activity from body surface potentials, has gained increasing clinical interest as a noninvasive imaging modality for underlying electrophysiological phenomena. We have previously presented an approach using 1) a transmural regularization to improve the joint reconstruction of electrical potentials on both the inner and outer surface of the ventricles; and 2) a nonlinear low-order dynamic spline-based parameterization to provide temporal regularization. This approach was tested for localizing endocardial pacing locations obtained from healthy hearts during catheter-based stimulation, using imprecise thorax geometry derived from limited computed tomographic scans. Results were promising, but the reconstructed solutions were overly smooth in space and time. Recently, L1-norm based spatial sparsity methods such as total-variation regularization have been reported to return more realistically sharp solutions in cardiac electrical imaging. In this paper, we compare and evaluate the performance of L2-norm based Tikhonov and L1-norm based total-variation regularization in conjunction with the spline parameterization and the transmural regularization. Numerical experiments were conducted on three subjects, each with multiple (*$\sim 20$*) endocardial pacing sites and evaluated against true pacing locations reported by the CARTO catheter mapping system. Variability was observed in the performance of the two methods across both pacing sites and subjects. However, the dependence of the results on subjects and ventricular pacing locations suggests that there is some correlation between the results and the specific geometry in each case. In our future work, we will investigate the approach of automatically inferring an optimal regularization norm from the data rather than fixing it a priori.*

## 1. Introduction

Electrocardiographic imaging (ECGI) is a technology to reconstruct of cardiac electrical activity from body surface potentials which has gained increasing clinical interest as a noninvasive modality for imaging underlying electrophysiological phenomena. However, it solves an ill-posed problem and small perturbations in the measurements can produce big variations in the solutions. To overcome this issue, it is necessary to introduce prior knowledge about the solution in the form of regularization. This can be in the form of spatial or temporal constraints.

In time, one approach is to to characterize the temporal behavior of the signal and impose it on the solution [1–3]. With that objective, we developed a method that nonlinearly approximates the multi-electrode ECG signal in time to reduce the presence of noise [4].

In space, the prior is often introduced in the optimization objective which solves a least-squares (LSQ) minimization between the ECG measurements ($y(t)$) and the unknown heart potentials ($x(t)$), plus a regularization term ($r(x)$) that introduces the prior knowledge (Equation 1).

$$\min_{x(t)} \|y(t) - Ax(t)\|_2^2 + \lambda r(x(t)) \tag{1}$$

This optimization problem, balances the LSQ fitting term against the regularization term $r(x(t))$. Which type of prior is being added depends on the function $r(\cdot)$, and with how much weight depends on the regularization parameter ($\lambda$) —typically chosen with the L-curve method [5].

There has been plenty of research in the ECGI community to determine what is the best regularization term to describe the electrical activation on the heart [6]. A classical approach is to use Tikhonov regularization [5]. In it, the regularization term is the L2 norm of the solution times the regularization matrix (R) (Equation 2).

$$\min_{x(t)} \|y(t) - Ax(t)\|_2^2 + \lambda \|Rx(t)\|_2^2 \tag{2}$$

In this approach, the regularization matrix describes the spatial characteristic to be minimized. Typical options are identity (0th order Tikhonov), gradient operator (1rst order) and Laplacian operator (2nd order). Another regularization approach that has gained lots of recent attention and which has shown to provide sparser solutions is Total Variation [7–9]. In this method the regularization term is an L1 norm of the spatial gradient of the solution (Equation 3).

$$\min_{x(t)} \|y(t) - Ax(t)\|_2^2 + \lambda \|Dx(t)\|_1 \quad (3)$$

There has been work in comparing spatial approaches in static solutions in time [9]. However, very few studies looked at the the effects of joined spatial and temporal regularization [10]. Here we compare two competing approaches for spatial regularization combined with a single non-linear temporal regularization approach. We test them in their capacity to estimate the point of earliest activation on the heart when applied to human data from pacing experiments.

## 2. Methods

In this work we use inverse methods that result from the combination of one of two spatial regularization approaches and a temporal regularization.

In space, the two approaches are modifications of the classical LSQ objective to minimize the gradient of the potentials on the heart in the regularization term. To approximate the gradient, both approaches use an operator that estimates the volumetric gradient at each point on the heart using a weighted neighborhood around it. The special characteristic of this gradient operator is that it includes nodes across the wall to determine the neighborhood, and thus does joint endocardial/epicardial regularization in surface geometries [4]. The difference between the two approaches that we compare is the measure used to calculate the norm of the gradient:

- **L1 norm:** This norm consists of the sum of absolute values and it is often used to solve inverse problems where the solutions are expected to be sparse [11].
- **L2 norm:** This norm is the classical sum of squares or euclidean norm and, counter to L1, this norm tends to produce smooth solutions.

In time, we use a temporal characterization of the signal that uses a multi-dimensional spline interpolation [4]. This method jointly approximates the potentials in all electrode with a B-spline function. To do so, it ignores the time stamps of the measurements and automatically determines a ”time-warp” that replaces them. With this approach, this method iteratively finds the best set of knot points $K_y$ and their temporal mixing $s(t)$ to fit the ECG recordings ($y(t)$) with the interpolation (Equation 4).

$$y(t) = K_y s(t) \quad (4)$$

Note that the knot points $K_y$ are themselves potential distributions on the torso and thus it is possible to calculate their corresponding potentials on the heart $K_x$ with any inverse method. With this reconstructed knot points, one can approximate the temporal sequence of potentials on the heart ($x(t)$) with the same temporal mixing obtained while fitting the ECG (Equation 5).

$$x(t) = K_x s(t) \quad (5)$$

Since the ground truth of the data is the localization of the first activation on the heart, we estimate the activation times from the potentials. To do so, we calculate the minimum $dv/dt$ —maximum negative slope— weighted by the norm of the gradient to favor activation times that capture the wavefront behavior of the solutions. Afterwards, we smooth the resulting activation times to reduce the error in the estimation.

## 3. Experiments and Results

We applied both methods to recordings from the same pacing experiment on human subjects. This datasets consists of ECG measured with 120 leads from 3 volunteers with healthy ventricles during a ventricular pacing procedure with a catheter device. The experiments were carried out with appropriate human subject permission from Charles University Hospital in Prague, Czech Republic, and in conjunction with standard atrial ablation procedures. Details of the experiment can be found in [4] and are summarized below.

During the intervention both ventricles were paced at different locations while the position of the catheter was recorded at the moment of pacing with the CARTO XP electroanatomical mapping system. The heart geometries —comprised the endocardium and the epicardium— were extracted from axial CT scans covering a section of the torso around the heart. A generic torso geometry was fitted to this limited view of the CT scans of each subject. With those geometries we generated the forward matrices using the open-source BEM solver within SCIRUN [12]. Then we used the forward matrix and the ECG recordings of each subject to reconstruct the corresponding EGM on the heart.

Finally, we estimated the activation times of each inverse solution and determined the site of earliest activation. To evaluate the solutions we then compared the obtained locations with the CARO coordinates projected onto the nearest node of the heart. The results for both methods are reported in Figure 1, Figure 2 and Figure 3 as the norm of the localization error (in mm).

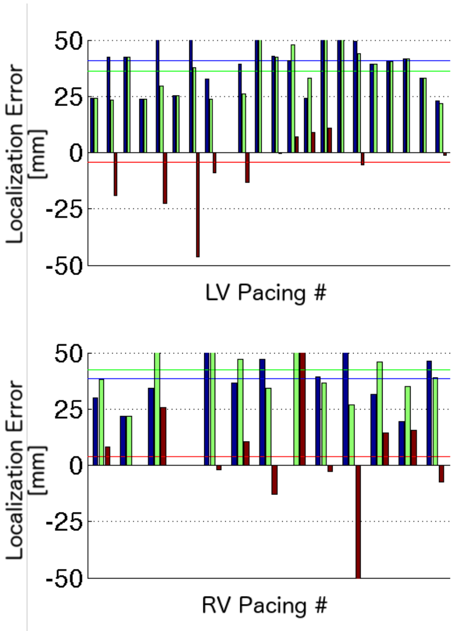


Figure 1: Localization error results for Subject 1. Bars in blue error for L1 norm, in green error for L2 norm and in red their difference (L2-L1). The lines are the averages across all pacing sites for L1 norm (blue), L2 norm (green) and difference (red).

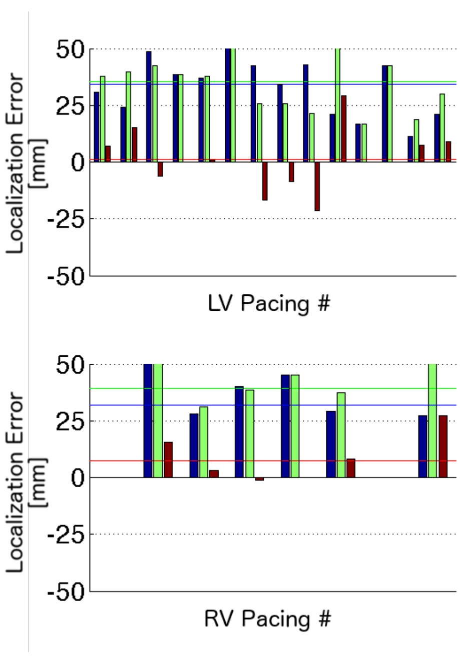


Figure 2: Localization error results for Subject 2. Bars in blue error for L1 norm, in green error for L2 norm and in red their difference (L2-L1). The lines are the averages across all pacing sites for L1 norm (blue), L2 norm (green) and difference (red).

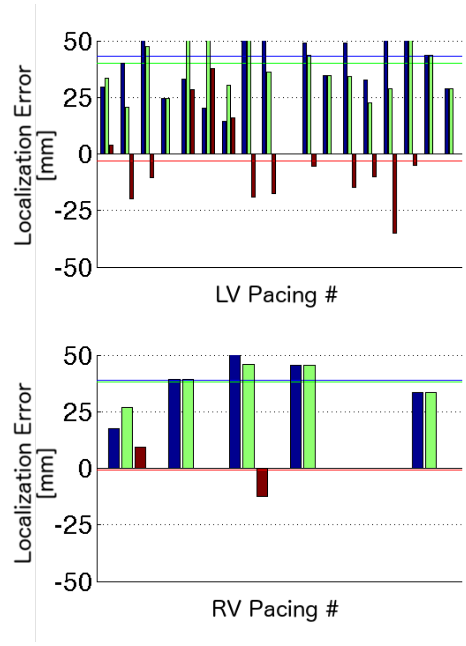


Figure 3: Localization error results for Subject 3. Bars in blue error for L1 norm, in green error for L2 norm and in red their difference (L2-L1). The lines are the averages across all pacing sites for L1 norm (blue), L2 norm (green) and difference (red).

## 4. Discussion

At first glance, the results seem to have random variability. However, there is some structure in them.

Within each subject, there is a wide spread of solutions: in some pacing sites we do fairly well with error $\sim 25$ mm, while for others we obtain errors higher than $50$ mm. The source of this variance across pacing sites is still unknown, but previous work showed some correlation with the proximity of the pacing site with respect to the septum [4].

Upon comparison of L1 and L2 regularization, the variability remains. For most pacing sites, the results in both are comparable, but there are some cases for which the difference between the two increases considerably. In the majority of the cases this difference favors L2 solutions. However, L2 norm does not dominate for all perspectives. This trend is reversed for subject 2 as well as for pacing sites on the RV for all subjects, for which L1 has on average more accuracy than its counterpart. Similar intersubject variations were observed in previous work using this dataset [13,14]. Those studies combined with the findings reported here suggest that there are geometry errors, variable across subjects, that have a different effects in the various modifications of the inverse solver.

Further research is necessary to be able to discern when it is advisable to use either approach to constraint the solutions. One option is to explore the variability across multiple recordings of pacings at the same location to determine if their spread shows any indication of better performance in L1 or L2. Another different approach is to investigate an automatic method to infer an optimal regularization norm from the data rather than fixing it a priori [15].

## 5. Conclusions

In this work we have studied the performance of L1 versus L2 norm regularization of the gradient in a joint spatial and temporal regularization algorithm for ECGI. The results are too variable to determine whether L1 or L2 is best for this application. However, they show some biasses across subjects and between ventricles that suggest that there is some structural characteristics that favor one type of regularization or the other and which could be exploited to improve accuracy in inverse solutions.

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Address for correspondence:

B-spiral group in the ECE dept. at Northeastern University
360 Huntington Ave., Boston, Massachusetts 02115