CS 170 Homework 6

Due **Monday 3/2/2026, at 10:00 pm (grace period until 11:59pm)**

Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, explicitly write "none".

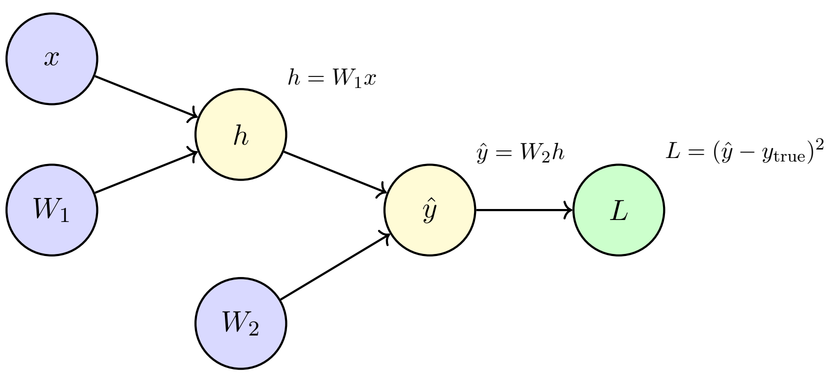
1 Backpropagation (Solo Question; 8 points)

In this problem, you will work out backpropagation on a couple of example networks.

(a) (4 points) Consider a 2-layer linear neural network with input x and weights W_1, W_2 given by

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$W_1 = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$
$$W_2 = \begin{pmatrix} 2 & 1 \end{pmatrix},$$

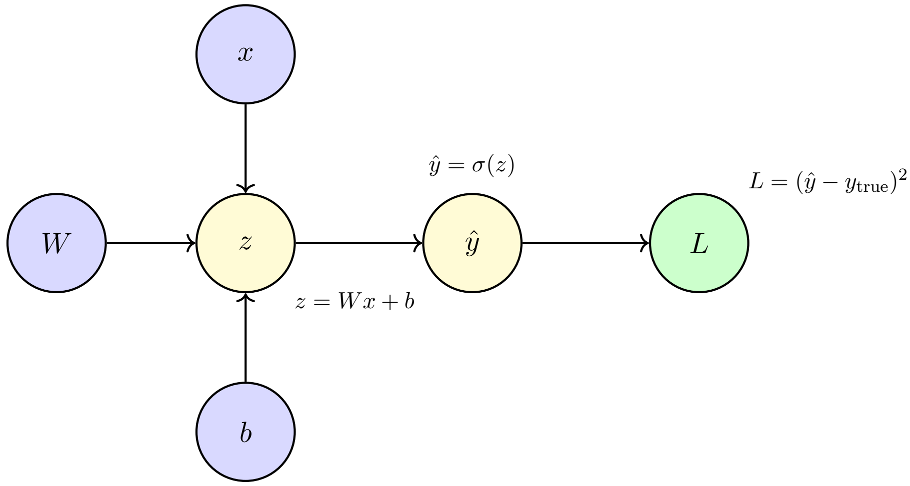
where the network has hidden layer $h = W_1 x$ and output $\hat{y} = W_2 h$. Assume the target value is $y_{\text{true}} = 15$. Perform forwards then backpropagation to compute the partial derivatives $\frac{\partial L}{\partial x}$, $\frac{\partial L}{\partial W_1}$, $\frac{\partial L}{\partial W_2}$ of the loss $L = (\hat{y} - y_{\text{true}})^2$.



(b) (4 points) Now consider a 1-layer network with sigmoid activation, with input x, weights W and bias b given by

$$x = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$
$$W = \begin{pmatrix} 2 & 1 \end{pmatrix}$$
$$b = -3,$$

where the network has output $\hat{y} = \sigma(Wx + b)$. Recall here the definition of the sigmoid function $\sigma(t) = 1/(1 + e^{-t})$. Assume the target value is $y_{\text{true}} = 5$. Perform forwards then backpropagation to compute the partial derivatives $\frac{\partial L}{\partial x}$, $\frac{\partial L}{\partial W}$, $\frac{\partial L}{\partial b}$ of the loss $L = (\hat{y} - y_{\text{true}})^2$.



Aside: Can this network ever achieve zero loss? Why or why not?

2 Polynomial Multiplication (10 points)

In this question, you will work out an example of polynomial multiplication; try to work out the answers by hand to gain intuition.

(a) (2 points) Let $h(x) = x^3 + 4x^2 + 5x + 2$. Evaluate $h(x)$ at the points $x = 1, i, -1, -i$.

(b) (2 points) Let $f(x) = x^2 + 3x + 2$ and $g(x) = x + 1$. Evaluate $f(x)$ and $g(x)$ at the points $x = 1, i, -1, -i$.

(c) (2 points) For each point $x = 1, i, -1, -i$, compute the pointwise product $f(x)g(x)$, and compare your answer to $h(x)$ from part (a).

(d) (4 points) Can you explain what you observed comparing parts (a) and (c)? And how does it relate to the Fourier transform?

3 Polynomial From Roots (10 points)

Consider the polynomial

$$p(x) = \prod_{i=1}^{n} (x - r_i)$$

with distinct roots $r_1, \ldots, r_n$. Recall that r is called a root of a polynomial $p(x)$ if $p(r) = 0$. Expanding the product above, we can write $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$. Give an algorithm to compute the coefficients $a_0, \ldots, a_n$. Show that your algorithm runs in time $O(n \log^2 n)$. For example, if $n = 2$, $r_1 = 1$ and $r_2 = 2$ then $a_2 = 1$, $a_1 = -3$, $a_0 = 2$.

4 Signal Processing (8 points)

Re-read the box on page 66 of the textbook (DPV), which explains how signal processing relies on the same mathematical foundations as polynomial multiplication. In particular, it describes how any linear time-invariant (LTI) system is fully characterized by its "impulse response", and how computing the systems output boils down to multiplying two polynomialsthat is, performing a convolution.

Look up (using LLMs, online lecture notes, etc.) how these concepts form the basis of digital signal processing (DSP) and how they are used in real-world applications (e.g., in audio engineering, medical imaging, telecommunications, or seismology). Then write a paragraph explaining your understanding of why the Fast Fourier Transform (FFT) is essential for enabling these applications in practice.