An alternative index for the contribution of precipitation on very wet days to the total precipitation

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ABSTRACT

Daily precipitation series from more than 1800 stations across Europe are analysed for the fraction of the total precipitation due to very wet days, i.e. days with precipitation amounts exceeding the 95th percentile. This fraction is calculated on a seasonal (three-monthly) basis for the period 1961-2010. A new index S95pTOT is introduced as an alternative to the frequently used index R95pTOT. Contrary to R95pTOT, which uses a fixed climatological 95th percentile, the new index assumes a separate 95th percentile for each year. Based on a Weibull distribution fit to the wet-day precipitation amounts, an analytical expression for S95pTOT is derived. It is shown that R95pTOT is strongly influenced by changes in the mean wet-day precipitation, whereas S95pTOT is more representative of changes in the distributional shape. The results for S95pTOT do not support the conclusion of a disproportional increase of extreme precipitation over northern Europe as was concluded from the trend in R95pTOT in earlier studies. Also the contrast between trends in northern and southern Europe in winter is less pronounced for S95pTOT than for R95pTOT.

1. Introduction

Now that consensus builds up on how the rise of the global temperature affects the intensity of the hydrological cycle (see e.g. Trenberth 2011), also the interest in the changes in seasonal mean precipitation and heavy precipitation is growing. Most information on the changes in precipitation refers to the seasonal mean precipitation, but when it comes to societal impacts, the changes in heavy precipitation are more relevant (IPCC 2012). The question that arises is how the changes in heavy precipitation relate to those in the mean precipitation. Changes of heavy precipitation relative to the mean have been studied from observations as well as climate model simulations for different regions, using a variety of methods. In order to separate changes in extreme precipitation from those in the mean, many studies consider the portion of the annual or seasonal precipitation total contributed by days with precipitation amounts exceeding some high level. The level is usually adapted to the probability distribution of daily precipitation amounts for the location of interest. This yields a standardised measure, suitable to compare locations with different precipitation regimes.

The index R95pTOT, recommended by the Expert Team on Climate Change Indices ETCCDI (Nicholls and Murray 1999; Zhang et al. 2011), is an example of such a measure. This index is used in many regional (Peterson and Manton 2008) and global (Frich et al. 2002; Alexander et al. 2006) studies. In the definition of R95pTOT, the 95th percentile of wet-day precipitation amounts, calculated for a chosen reference period, is used to identify very wet days. The sum of daily precipitation amounts exceeding this level is divided by the total precipitation to obtain the fraction of precipitation on very wet days. Trends in R95pTOT have been analysed by Klein Tank and Können (2003) for the period 1949-1999 for 151 stations across Europe on an annual basis. They found a significant positive trend in R95pTOT for a substantial part of those stations for which also a significant positive trend in the annual totals was found. They concluded that this supports the notion of an amplified response of the extremes relative to the precipitation total. In more recent literature R95pTOT is also considered (Turco and Llasat 2011; Ducić et al. 2012; Sillmann et al. 2013; Donat et al. 2013) as an indicator of the amplified response of extreme precipitation events to climate change.

A disadvantage of the index R95pTOT at a seasonal scale, reported by Zolina et al. (2009), is that the number of days falling into the highest category can vary strongly over time or become zero in seasons for which the highest precipitation amount does not exceed the 95th percentile of the reference period. They introduced an alternative index, resembling R95pTOT in its definition, but based on the Gamma distribution for the wet-day amounts and the associated theoretical distribution of the fractional contribution of the wettest days to the seasonal or annual total. This new index shows less year-to-year variation, although it is essentially similar to the procedure of Klein Tank and Können (2003) for R95pTOT.

In this paper, we address an additional disadvantage of R95pTOT as an indicator of an amplified response of extreme precipitation, namely that a change in the mean *without* a change in the shape of the distribution also affects R95pTOT. Hence, a trend in R95pTOT not necessarily represents a change in the distributional shape associated with an amplified response of extreme precipitation (Section 3). We introduce an index S95pTOT, which does not have this disadvantage and also shows less variation over the years than R95pTOT. The fundamental difference between R95pTOT and S95pTOT is that in the latter no fixed level is used to define the very wet days. Instead, this level is allowed to vary by year and season. In addition, an analytical expression is derived for S95pTOT in terms of the shape parameter of a fitted Weibull distribution. The 95th percentile is no longer explicitly needed, but is instead an implicit part of this expression. Because the analytical expression for S95pTOT is derived in terms of the Weibull shape parameter, the association between S95pTOT and the distributional shape follows naturally. The trends in the new index are compared with those of the traditional index for more than 1800 stations across Europe. The central question is whether the trends of S95pTOT also support the earlier findings of a disproportional change of the extremes over Europe.

The remainder of the paper is structured as follows. Section 2 describes the data used in this study. In Section 3 we argue why R95pTOT is suboptimal as an indicator of a disproportional change of the extremes relative to the mean and define S95pTOT. Results of tests for trend in the index time series are presented in Section 4. The final section presents the conclusion and some final remarks concerning the new index.

2. Precipitation data

The daily precipitation data used in this study was obtained from the European Climate Assessment dataset (ECA&D, Klein Tank et al. 2002). The so-called 'blended' station series in ECA&D covering the study period 1961-2010, were initially selected. All these series were tested for homogeneity, using the methods of Wijngaard et al. (2003), and labelled as 'Useful' for the somewhat longer test period 1951-2011. Homogeneity tests for this period are part of the standard quality control of ECA&D. We do not expect the homogeneity of the series for the study period to be substantially different. From the selected series those series were rejected for which less than 80% of the data was available over the study period. The resulting set of stations varies in size through the year between 1815 stations in JJA to 1856 stations in MAM. Data from ECA&D were also used by Zolina et al. (2009). That study was restricted to the original, non-blended series to rule out potential artefacts introduced by the blending process applied within ECA&D. In this process, time series are filled up with data from neighbouring stations, where possible, and augmented with synoptical data, distributed through the Global Telecommunication System up to the current month. However, the quality control procedures and the homogeneity checks in ECA&D, to which only the 'blended' station series are subjected, substantially reduces the risks of potential artefacts.

3. Method

a. Background of the used indices

The index R95pTOT is defined for a season within a specific year as the ratio of the sum of all daily precipitation amounts exceeding the climatological (1961-1990) 95th percentile Q of the wet-day precipitation amounts to the total precipitation (see also the full definition by Klein Tank and Können 2003). Wet days are here defined as those days with $\delta = 1$ mm of precipitation or more.

Let g be the probability density of *all* daily precipitation amounts (including those on dry days), μ the corresponding mean and N the number of days in the season of interest. The sum of the precipitation amounts exceeding Q can then be approximated by $N \int_Q^\infty x g(x) dx$ and the total precipitation by $N\mu$, which leads to the following approximation of R95pTOT:

$$\text{R95pTOT} \approx \frac{1}{\mu} \int_Q^\infty x g(x)\, dx \ . \tag{1}$$

This can be rewritten in terms of the probability density g_w and mean μ_w of the wet-day precipitation amounts as

$$\text{R95pTOT} \approx \frac{1}{\mu_\text{w}} \int_Q^\infty x\, g_\text{w}(x)\, dx \tag{2}$$

(see Appendix A). Because Q is kept constant, a change in R95pTOT may be due to a change in the mean rather than a change in the shape of the distribution. This is best illustrated by introducing the standardised precipitation amount $x' = x/\mu_\text{w}$ as integration variable in Eq. 2:

$$\text{R95pTOT} \approx \int_{Q/\mu_\text{w}}^\infty x'\, g'_\text{w}\left(x'\right)\, dx' \ , \tag{3}$$

where $g'_\text{w}(x') = \mu_\text{w} g_\text{w}(x)$ is the density of x'. Now assume that g'_w does not change over time, whereas μ_w does. The latter induces a change in R95pTOT (to a first order) of

$$\Delta\text{R95pTOT} \approx \left(\frac{Q}{\mu_\text{w}}\right)^2 g'_\text{w}\left(\frac{Q}{\mu_\text{w}}\right) \frac{\Delta\mu_\text{w}}{\mu_\text{w}} \ . \tag{4}$$

The change in R95pTOT induced by the change in μ_w is thus to a first order proportional to the relative change in μ_w. Though Q refers to the climatological 95th percentile over some reference period, it may not correspond with the 95th percentile in an individual season, and therefore neither does R95pTOT represent the contribution of the upper 5%. Therefore, we introduce a modified index RS95pTOT based on the time-varying 95th percentile q for the season of interest. Analogous to Eqs. 2 and 3, this index can be written as

$$\mathrm{RS95pTOT} \approx \frac{1}{\mu_\mathrm{w}} \int_{q}^{\infty} x\, g_\mathrm{w}(x)\, dx \;=\; \int_{q/\mu_\mathrm{w}}^{\infty} x'\, g'_\mathrm{w}(x')\, dx' \;. \tag{5}$$

In contrast to R95pTOT, a change in the mean does not alter the integral as long as g'_w does not change, because then q/μ_w remains unchanged.

We now assume that the daily precipitation amounts X on wet days follow a two-parameter Weibull distribution with scale parameter α, shape parameter c, shifted by the wet-day threshold δ. The corresponding distribution function is given by

$$G_\mathrm{w}(x) \;\equiv\; \Pr(X \leq x) \;=\; 1 \;-\; \exp\left[-\left(\frac{x-\delta}{a}\right)^{c}\right] \;, \qquad\qquad x \geq \delta \tag{6}$$

and the probability density by

$$g_\mathrm{w}(x) \;\equiv\; \frac{dG_\mathrm{w}}{dx} \;=\; \frac{c}{a}\left(\frac{x-\delta}{a}\right)^{c-1} \exp\left[-\left(\frac{x-\delta}{a}\right)^{c}\right] \;. \tag{7}$$

The expression for RS95pTOT then becomes (Appendix A)

$$\mathrm{RS95pTOT} \approx \frac{a\Gamma\left(\frac{1}{c}+1\right)}{\delta \,+\, a\Gamma\left(\frac{1}{c}+1\right)} \left[\frac{0.05\,\delta}{a\Gamma\left(\frac{1}{c}+1\right)} \;+\; 1 \;-\; P\left(\frac{1}{c}+1, -\log(0.05)\right)\right] \;, \tag{8}$$

with $P(\cdot,\cdot)$ the normalised lower incomplete Gamma function (Abramowitz and Stegun 1965, Eq. 6.5.1) and $\Gamma(\cdot)$ the complete Gamma function (Abramowitz and Stegun 1965, Eq. 6.1.1.). Eq. 8 can be simplified considerably by taking the limit as $\delta \to 0$, yielding the new index proposed in this paper,

$$\mathrm{S95pTOT} \;=\; 1 \;-\; P\left[\frac{1}{c}+1, -\log(0.05)\right] \;. \tag{9}$$

S95pTOT can be seen as an approximation of RS95pTOT for small δ and has the advantage of depending only on the Weibull shape parameter. The factor in Eq. 8 preceding the square brackets is the ratio between the mean excess over the wet-day threshold and the mean wet-day precipitation, which is very close to one for δ sufficiently small. The first term within the square brackets equals 0.05 times the wet-day threshold divided by the mean excess, which is very small. Assuming a shifted Gamma distribution for the wet-day precipitation amounts, an expression similar to Eq. 9 can be derived, depending on the Gamma shape parameter only. The preference for the Weibull distribution is motivated in Appendix B. by means of an L-moment ratio diagram.

Alternatively, RS95pTOT and S95pTOT can be estimated empirically, without assuming a distribution. The index RS95pTOT can be calculated as the sum of the n_q largest wet-day precipitation amounts divided by the sum over all n wet-day amounts, where n_q equals 0.05 n, rounded to the nearest integer. The empirical estimate of S95pTOT can be obtained similarly, except that in this case all wet-day precipitation amounts should be reduced by δ beforehand.

b. Application to station data

For each station (three-monthly) seasonal time series of the original index R95pTOT and the new index S95pTOT for the period 1961-2010 were analysed. The series of RS95pTOT were also studied to assess the effect of neglecting δ. The series of R95pTOT were obtained from ECA&D to ensure correspondence with earlier studies based on this index. The series of RS95pTOT and S95pTOT were calculated using the empirical as well as the parametric approach. For the parametric estimates, a two-parameter Weibull distribution was fitted to the shifted wet-day amounts (over δ) using probability weighted moments (Boes et al. 1989; Hosking and Wallis 1997). Only seasons with 80 or more days with data and 10 or more wet days (δ=1 mm) were taken into account, to prevent the empirical estimate from becoming zero. The estimation of S95pTOT was tested with synthetic data from a known Weibull distribution, as described in Appendix C. The estimates of S95pTOT were found to be nearly unbiased. Furthermore, estimating S95pTOT parametrically rather than empirically reduced the uncertainty.

To detect trends in the time series of R95pTOT, RS95pTOT and S95pTOT the Mann-Kendall statistic MK (Yue et al. 2002a) was determined for each station and each season. In the theoretical variance of MK the possibility of ties is taken into account. The potential influence of a non-zero lag-1 autocorrelation is dealt with by trend-free pre-whitening as discussed by Yue et al. (2002b).

Apart from testing the statistical significance of the MK statistic for each station individually, the significance of the percentage r_+ of stations with a positive value of the MK statistic was tested. This field significance was determined using a block permutation procedure with a blocksize of three years to preserve the dependence between successive years. The seasonal values of R95pTOT, RS95pTOT and S95pTOT were permutated for all stations simultaneously to preserve the spatial dependence. The latter can have a considerable influence on the variance of r_+ (cf. Douglas et al. 2000). This permutation procedure is similar to the moving block bootstrap in Kiktev et al. (2003), except that the blocks are sampled without replacement, so every sample consists of a random permutation of blocks. In contrast with the bootstrap, the permutation procedure preserves the sample size for stations with missing data. For each permutation replication the percentage r_+^* of stations was determined for which MK > 0, and this number was compared with the value r_+ for the observations. Let f be the fraction of permutation replications for which $r_+^* > r_+$. A two-sided significance probability was obtained as $p = 2\min(f, 1-f)$.

4. Results

Figure 1 shows the climatological mean (1961-2010) of R95pTOT (top) and S95pTOT determined either empirically (center) or based on the Weibull distribution (bottom) in the DJF season. It is seen that the mean of S95pTOT is somewhat higher than that of R95pTOT, and that this difference is generally larger than the difference between both versions of S95pTOT. The maps give no indication of a bias resulting from the Weibull assumption. Figure 2 shows the relative standard deviation of R95pTOT and both versions of S95pTOT, expressed as a percentage of the mean. The standard deviation was determined here using the mean square of successive differences in order to rule out the influence of trends. The standard deviation of R95pTOT is larger than that of S95pTOT. This reflects the strong interannual variation of R95pTOT, partly related to variations in the mean wet-day precipitation (see Eq. 4). The values of S95pTOT based on the Weibull distribution are slightly less variable than those determined empirically.

Figure 3 shows the sign and the significance of the trend in R95pTOT (top) and S95pTOT (bottom) for the DJF season. Trends in RS95pTOT (not shown here) reveal a pattern very similar to that of S95pTOT. While particularly the northern part of Europe is dominated by a (significant) increase of R95pTOT, this is far less pronounced for S95pTOT. At many sites the signs of the trend in S95pTOT and R95pTOT differ. Insignificant (positive or negative) trends become significantly negative and significant positive trends are reduced to insignificant trends. This is particularly striking in southern Scandinavia, the Netherlands, Germany and the UK. In Spain and southern France the number of stations with a negative trend is less for S95pTOT than for R95pTOT. As a whole, the contrast between the trends in northern and southern Europe is more diffuse for S95pTOT than for R95pTOT.

Table 1 displays the percentages of the stations for which a positive trend was found in the indices S95pTOT, RS95pTOT and R95pTOT, the fraction of wet days f_{w}, the mean wet-day amount μ_{w}, and the coefficient of variation of wet-day precipitation CV_{w}. A distinction is made between the stations north and south of the 48th parallel. For each percentage the field significance (based on the block permutation procedure described in Subsection 3b, with 1000 permutation replications) is also listed.

In DJF and MAM the percentages of positive trends in R95pTOT clearly indicate an increase for the majority of the northern stations. This majority is field significant. The same holds for the corresponding percentages for μ_{w}. The percentage of positive trends in S95pTOT and RS95pTOT for northern Europe in the DJF and MAM seasons is substantially lower than for R95pTOT, in many cases even below 50%. For the southern stations the percentage of positive trends in R95pTOT is smaller than for the northern stations in those seasons and not field significant. Consistently, no field significant trend is found for μ_{w}. These results confirm the suggested influence of changes in μ_{w} on R95pTOT. The percentages of positive trends in R95pTOT for the southern stations are of the same order as those in S95pTOT and RS95pTOT in DJF and MAM. Though the trends in R95pTOT are not field significant for these stations, the trends in S95pTOT and RS95pTOT are in a number of cases. Finally, it is observed that in general the percentage of positive trends in S95pTOT and RS95pTOT corresponds better with that in CV_{w} than that in μ_{w}.

A large contrast is observed between northern and southern Europe regarding the changes in the fraction of wet days in the seasons DJF and JJA. The majority of the northern stations shows an increase, whereas for the majority of the southern stations a decrease is found. The latter is field significant. In MAM an overall decrease is seen whereas in SON most stations in northern and southern Europe show an increase.

Differences between the results for S95pTOT and RS95pTOT in Table 1 reflect the influence of the finite wet-day threshold δ. These differences are generally small compared to the differences with the results for R95pTOT. Also (not shown here) the spatial pattern of the MK statistics for RS95pTOT is very similar to that for S95pTOT, shown in the lower panel of Fig. 3 for the DJF season.

In particular for the northern part of Europe it is likely that the results of Table 1 are predominated by the trends in regions with a high station density in Fig. 3. It is interesting to zoom in on these regions. We defined two subregions, 'NorSwe' between 55° and 65°N and between 5° and 27.5°E, containing southern Norway, Sweden and a minor part of Finland, and 'NetGer' between 47° and 55°N and between 3° and 15°E, covering primarily the Benelux and Germany.

Figure 4 shows the same as Fig. 3 but zooms in on the subregions NorSwe (A,C) and NetGer (B,D). For many stations within both regions the trends in S95pTOT and R95pTOT have opposite signs (increasing R95pTOT in panels A and B versus decreasing S95pTOT in panels C and D), though fewer significant trends are counted for S95pTOT than for R95pTOT. For NorSwe the most striking are the Norwegian stations near the coast, some of which have significant opposite trends for R95pTOT and S95pTOT. In subregion NetGer, the trends in the Netherlands stand out. The trend in R95pTOT is in general positive throughout the country, significant near the coast and near the southern border. For S95pTOT the trend is on the whole negative (even significantly so at some eastern stations) except for the coastal area.

In analogy with Table 1, Table 2 summarises the results for the two subregions. Given the dense clusters of stations in both subregions, the results in Table 2 may strongly influence those for northern Europe in Table 1. Large percentages of stations with an increase in R95pTOT are found in DJF and MAM for the NetGer subregion, as well as in JJA for the NorSwe subregion. These increases again appear to be related to the increases in μ_{w}. For DJF, the majority of the stations in the NorSwe subregion show a decrease in CV_{w} and S95pTOT, which is field significant for the empirical estimate of S95pTOT. The effect of this decrease on R95pTOT is compensated by an increase in μ_{w}, found at 82.6% of the stations in this subregion. The increase of f_{w} in DJF in northern Europe can, at least partly, be ascribed to region NorSwe, while the decrease in MAM and the slight increase in SON are accounted for by NetGer. None of the changes in f_{w} are field significant, as was also concluded for the whole of northern Europe, where these changes are partly averaged out.

Figure 5 gives an example of how the values of R95pTOT and S95pTOT compare for an individual station. The station Enonkoski Simanala (Finland) was chosen because of the high MK statistic found for the index R95pTOT in the DJF season. Transformed to the standard-normal scale, this value was 4.08 (highly significant rise) and the non-parametric estimate of the slope (Sen 1968) amounted to 4.5%/decade. On the contrary, the index S95pTOT calculated from the same daily data using fitted Weibull distributions resulted in an MK statistic of 1.43 (not significant at the 5%-level) and a slope of 0.6%/decade. Both index series are displayed in the upper panel of Fig. 5. The first half of the series of R95pTOT contains a number of zeroes. These correspond with seasons in which the highest daily precipitation amounts fail to exceed the climatological 95th percentile. The zeroes do by definition not occur in the S95pTOT series. Apart from that effect, the year-to-year variations are smaller for the S95pTOT series than for R95pTOT (also seen in Fig. 2), enhancing the detection of a trend if present. The lower panel panel of Fig. 5 shows the difference R95pTOT-S95pTOT (left ordinate) versus time together with the mean wet-day precipitation (right ordinate). The strong correspondence between both sequences confirms the conclusion drawn from the tables that the trend in μ_{w} can explain much of the difference between the trends in R95pTOT and S95pTOT.

5. Conclusion and discussion

The objective of this study was to assess changes in the relative contribution of very wet days (i.e. days with precipitation amounts exceeding the 95th percentile) to the total precipitation amounts. Particular attention was given to the detection of a disproportional increase of extreme precipitation amounts, relative to the total precipitation. The index R95pTOT has often been used to monitor such changes. However, the use of this index has been questioned because of its strong year-to-year variations (Zolina et al. 2009). In this paper it is shown that R95pTOT is also influenced by changes in the mean wet-day precipitation. An alternative index S95pTOT was introduced, exhibiting less year-to-year variation and being better suited to characterizing a disproportional change of precipitation extremes. Assuming the wet-day precipitation amounts are Weibull distributed, an analytical expression for S95pTOT was derived in terms of the Weibull shape parameter. For the quality-checked daily precipitation series of more than 1800 stations across Europe covering the period 1961-2010 the seasonal series of R95pTOT and S95pTOT were analysed. For all four seasons we compared the number of positive trends in R95pTOT and S95pTOT, the fraction of wet days, the mean and the CV of the wet-day precipitation amounts over the period 1961-2010.

The reason that R95pTOT is sensitive to the changes in the mean wet-day precipitation lies in the fact that a fixed, climatological 95th percentile is used to define very wet days. We argued that a trend in the mean wet-day precipitation then contributes in a positive sense to the trend in R95pTOT, which was not fully recognised in earlier interpretations of R95pTOT as a measure of 'disproportional change in the extremes'. An additional problem in the definition of R95pTOT is that year-to-year variations in the number of wet days and mean wet-day precipitation induce strong variations and zeroes in this index. These can also affect the observed trend in R95pTOT.

The proposed index S95pTOT is based on a 95th percentile that is not assumed constant over time. Evaluation of S95pTOT using the proposed analytical expression does not require an explicit estimate of the 95th percentile, but only the Weibull shape parameter. The contrast between the trends in northern and southern Europe was far less pronounced for S95pTOT than for R95pTOT. For many stations the trend in both indices even has an opposite sign. The net difference with the trend in R95pTOT was a decreased number of significant positive trends and a moderate increase in the number of negative trends. From the percentages of positive trends it is concluded that in Europe the trend in R95pTOT is related to the trend in the mean, whereas the trend in S95pTOT seems to be more influenced by the trend in the CV of the wet-day precipitation amounts.

The fact that the climatological 95th percentile over the reference period is not representative of the 95th percentile of an individual season is also the real cause behind the zeroes in the R95pTOT series reported by Zolina et al. (2009). Their parametric approach actually avoids these zeroes by tackling the discrete nature of the calculation of R95pTOT, so that in those cases R95pTOT becomes very small rather than zero. However, because they also obtain the 95th percentile from a reference period, their trends more or less resemble those of R95pTOT. In this study, rather than presenting an alternative calculation method, we have presented an alternative index with its own definition and interpretation.

For seasons in which either the number of valid daily precipitation amounts drops below 80 or the number of wet days falls below 10, S95pTOT was not calculated. This has no serious effects on the S95pTOT series for the northern stations. For Southern Europe, the lack of wet days reduces the number of stations for which S95pTOT is calculated in the JJA season by about 20%. This number does not show a distinctive trend, which suggests no systematic influence on the sign of trends in S95pTOT. However, missing values in the series of S95pTOT reduce the power of the tests for trend. For seasons having very few wet days, one might question the relevance and interpretation of such an index anyway. For seasons having sufficient wet days, S95pTOT clearly *is* meaningful.

Though R95pTOT is also undeniably a meaningful index, one should be careful with its interpretation. The 95th percentile determined over a climatological period identifies precipitation events that are perceived as ‘severe’ using the climatological period as a reference for a certain location. This makes R95pTOT very useful within a climate-change impact context. However, to infer if extremes increase disproportionally and characterise changes in the distributional shape, other measures are more appropriate. The index S95pTOT introduced here is one of them.

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APPENDIX A

Relative contribution of the upper 5% of the distribution

The density g is related to the density g_{w} of the wet-day precipitation amounts by $g(x) = f_{\mathrm{w}} g_{\mathrm{w}}(x)$ for $x \geq \delta$, where f_{w} is the fraction of wet days. Equation 1 can then be rewritten as

$$\mathrm{R95pTOT} \approx \frac{f_{\mathrm{w}}\mu_{\mathrm{w}}}{\mu} \left(\frac{1}{\mu_{\mathrm{w}}} \int_{Q}^{\infty} x g_{\mathrm{w}}(x)\, dx \right) . \tag{A1}$$

The factor $f_{\mathrm{w}}\mu_{\mathrm{w}}/\mu$ gives the fraction of the precipitation total contributed by the wet days. Even with a relatively high wet-day threshold δ of 1 mm, this factor is virtually equal to one. Equation A1 then results in Eq. 2.

Replacing x by $y + \delta$, Eq. 5 gives for RS95pTOT:

$$\mathrm{RS95pTOT} \approx \frac{\delta}{\mu_{\mathrm{w}}} \int_{q-\delta}^{\infty} g_{\mathrm{w}}(y+\delta)\, dy + \frac{1}{\mu_{\mathrm{w}}} \int_{q-\delta}^{\infty} y\, g_{\mathrm{w}}(y+\delta)\, dy . \tag{A2}$$

The integral in the first term on the right-hand side represents $\Pr(X > q) = 0.05$, because q denotes the 95th percentile of the distribution of X. Substituting for g_{w} the density of the shifted Weibull distribution (Eq. 7) in the second term on the right-hand side, we have

$$\begin{aligned} \frac{1}{\mu_{\mathrm{w}}} \int_{q-\delta}^{\infty} y\, g_{\mathrm{w}}(y+\delta)\, dy &= \frac{1}{\mu_{\mathrm{w}}} \int_{q-\delta}^{\infty} \frac{y}{a} \exp\left[-\left(\frac{y}{a}\right)^{c} \right] c \left(\frac{y}{a}\right)^{c-1} dy \\ &= \frac{a}{\mu_{\mathrm{w}}} \int_{\left(\frac{q-\delta}{a}\right)^{c}}^{\infty} u^{1/c} \exp(-u)\, du = \frac{a}{\mu_{\mathrm{w}}} \Gamma\left[\frac{1}{c} + 1, \left(\frac{q-\delta}{a} \right)^{c} \right] , \end{aligned} \tag{A3}$$

where $\Gamma(\cdot,\cdot)$ denotes the upper incomplete Gamma function (Abramowitz and Stegun 1965, Eq. 6.5.3). Since q is the 95th percentile of the shifted Weibull distribution (Eq. 6) with parameters a and c,

$$G_{\rm w}(q) \;=\; 1 \;-\; \exp\left[-\left(\frac{q-\delta}{a}\right)^{c}\right] \;=\; 0.95 \qquad \text{or} \qquad \left(\frac{q-\delta}{a}\right)^{c} \;=\; -\log(0.05)\;. \qquad \text{(A4)}$$

Combining Eqs. A2, A3 and A4 leads to

$$\text{RS95pTOT} \;\approx\; \frac{0.05\,\delta}{\mu_{\rm w}} \;+\; \frac{a}{\mu_{\rm w}}\Gamma\left[\frac{1}{c}+1,-\log(0.05)\right]. \qquad \text{(A5)}$$

Substitution of $\mu_{\rm w} = \delta + a\Gamma\left(\frac{1}{c}+1\right)$ finally yields

$$\text{RS95pTOT} \;\approx\; \frac{a\Gamma\left(\frac{1}{c}+1\right)}{\delta \;+\; a\Gamma\left(\frac{1}{c}+1\right)}\left[\frac{0.05\,\delta}{a\Gamma\left(\frac{1}{c}+1\right)} \;+\; 1 \;-\; P\left(\frac{1}{c}+1,-\log(0.05)\right)\right]\;, \qquad \text{(A6)}$$

where $P(\cdot,\cdot)$ refers to the normalized lower incomplete Gamma function (Abramowitz and Stegun 1965, Eq. 6.5.1) and $\Gamma(\cdot)$ refers to the complete Gamma function.

APPENDIX B

Choice of the Weibull distribution

The two-parameter Gamma distribution has been frequently used to model wet-day precipitation amounts (e.g. Groisman et al. 1999; Wilby and Wigley 2002; Zolina et al. 2009). For daily precipitation data from the U.S. Pacific Northwest, however, Duan et al. (1998) showed that the diagram of the sample L-Skewness versus the sample L-CV was more consistent with the theoretical relation between the L-Skewness and L-CV for the Weibull distribution than with that for the Gamma distribution. The L-CV and the L-Skewness are alternatives to the conventional coefficient of variation (CV) and coefficient of skewness, based on linear combinations of the ranked observations. These L-moment ratios are less biased and more robust to outliers than the conventional moment ratios. L-moment ratio diagrams have been used to select a probability distribution, in particular within the hydrological context. They were introduced by Hosking (1990) and their preference to conventional moment ratio diagrams was further demonstrated by Vogel and Fennessey (1993). The theoretical L-CV τ_2 and L-Skewness τ_3 for the two-parameter Weibull distribution can be obtained from Goda et al. (2010) as:

$$\tau_2 = 1 - 2^{-1/c} \qquad \text{and} \qquad \tau_3 = \frac{1 - 3 \cdot 2^{-1/c} + 2 \cdot 3^{-1/c}}{1 - 2^{-1/c}} \; . \tag{B1}$$

These expressions can be combined to obtain the relation

$$\tau_3 = \frac{1}{\tau_2} \left[1 - 3(1 - \tau_2) + 2(1 - \tau_2)^{2 \log 3} \right] \; . \tag{B2}$$

For the two-parameter Gamma distribution, as a special case of the Pearson type-III distribution, it follows from Hosking and Wallis (1997) that

$$\tau_2 = \frac{\Gamma\,(c + 1/2)}{c\sqrt{\pi}\,\Gamma(c)} \qquad \text{and} \qquad \tau_3 = 6 I_{1/3}(c, 2c) - 3 \; , \tag{B3}$$

where c is the Gamma shape parameter and $I_x(\cdot,\cdot)$ the normalized incomplete beta function (Abramowitz and Stegun 1965, Eq. 6.6.2). For the latter Hosking and Wallis (1997) also provide a rational-function approximation (not shown here), which was used to plot the relation between τ_3 and τ_2 for the Gamma distribution. In Fig. 6 the sample L-Skewness of the wet-day precipitation is plotted versus the sample L-CV for all stations and all individual DJF seasons, together with the theoretical curves for the Weibull and Gamma distributions. Note that the wet-day precipitation amounts were reduced by the wet-day threshold δ to comply with the zero lower bound of the two theoretical distributions. It can be seen that for these data the Weibull distribution performs at least as well as the Gamma distribution, though the skewness is still somewhat underestimated. The L-moment ratio diagrams for the other seasons are similar.

APPENDIX C

Testing the estimation of S95pTOT

To test the performance of the two estimators for S95pTOT, a set of 50 000 simulations was conducted. In each simulation 100 values were generated from a Weibull distribution with shape parameter $c = 0.6$ and scale parameter $\alpha = 1.0$. A sample was formed by accepting values at random with a probability of 60%. This incorporates the effect of a varying number of wet days in a season as a source of uncertainty. From each sample S95pTOT was approximated empirically, as well as through the shape parameter of a Weibull distribution fitted to the sample. Figure 7 compares the histograms of the two estimators with the theoretical value, based on the known Weibull shape parameter. The dashed vertical line signifies the theoretical value associated with the chosen shape parameter. From the histograms we can conclude that both estimators are nearly unbiased. Furthermore, the use of the Weibull shape parameter seems to narrow the histogram, compared to that of the empirical estimates, e.g. the width of the equitailed 90% interval reduces from approximately 28% for the empirical estimates to 19% for the estimates based on the Weibull fits. This is in line with the smaller interannual standard deviation for the Weibull-based estimates, displayed in Fig. 2.

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List of Tables

TABLE 1. Percentages of the stations for which the MK statistic indicates an increase for f_w, μ_w, CV_w, S95pTOT, RS95pTOT and R95pTOT. For S95pTOT and RS95pTOT the empirical estimate ('Emp.') is given as well as the parametric estimate through the Weibull distribution ('Par.'). The northern stations and the southern stations are separated by the 48th parallel. The numbers in parentheses are the number of stations on which the given percentages are based. Percentages that are field significant at the 5%-level (two-sided) are printed in bold. The p-values (percent) are shown on the next line, indicated with a 'p'.

			f_w	μ_w	CV_w	S95pTOT		RS95pTOT		R95pTOT
						Emp.	Par.	Emp.	Par.	
DJF	North		72.9	**84.8**	41.4	38.5	38.6	45.9	50.5	**77.4**
	[1445]	p =	48	1	20	10	14	59	84	4
	South		**7.1**	54.9	62.0	63.5	**69.6**	60.3	**71.1**	60.3
	[408]	p =	1	78	17	16	1	35	0	37
MAM	North		28.9	**78.6**	55.8	42.6	58.5	46.5	**64.5**	**74.6**
	[1447]	p =	47	4	21	38	10	79	2	1
	South		**16.7**	46.7	**66.1**	**63.6**	**69.3**	62.4	66.8	57.7
	[407]	p =	3	75	5	3	3	7	8	46
JJA	North		64.4	66.4	49.4	49.4	48.6	51.8	52.5	60.3
	[1448]	p =	94	17	99	80	99	86	60	20
	South		30.1	51.0	54.8	55.3	59.2	54.0	58.6	48.8
	[365]	p =	25	92	23	30	7	41	11	99
SON	North		66.7	47.0	59.6	**59.9**	57.5	**59.9**	57.9	55.0
	[1448]	p =	45	83	12	2	32	2	23	50
	South		73.8	44.8	59.2	51.7	57.7	55.0	60.6	47.5
	[404]	p =	38	75	11	80	19	57	7	90

TABLE 2. Similar to Table 1, but now for two selected subregions, 'NorSwe' (55°-65°N, 5°-27.5°E) and 'NetGer' (47°-55°N, 3°-15°E), as indicated in Fig. 4.

			f_w	μ_w	CV_w	S95pTOT		RS95pTOT		R95pTOT
						Emp.	Par.	Emp.	Par.	
DJF	NorSwe		90.8	82.6	36.5	**33.0**	34.9	38.5	42.0	66.7
	[436]	p =	16	6	7	4	8	17	22	17
	NedGer		58.0	90.4	39.8	35.7	34.8	45.8	51.6	85.2
	[742]	p =	83	6	38	23	22	70	87	12
MAM	NorSwe		53.1	72.4	51.0	41.8	53.1	45.5	55.6	67.4
	[435]	p =	75	11	77	31	56	67	33	11
	NedGer		4.3	83.7	59.0	41.0	63.2	46.0	**72.9**	**82.3**
	[742]	p =	11	8	28	59	14	88	3	2
JJA	NorSwe		72.0	**85.8**	55.0	54.1	57.3	60.8	**66.3**	**75.7**
	[436]	p =	83	0	39	54	27	7	5	0
	NedGer		60.2	55.0	48.2	48.7	45.3	49.6	46.4	52.2
	[742]	p =	94	84	91	80	77	86	83	85
SON	NorSwe		35.8	44.7	56.4	52.3	59.2	52.5	60.6	55.3
	[436]	p =	73	81	37	42	27	32	13	26
	NedGer		89.9	43.1	65.0	**69.0**	58.8	**68.6**	57.1	54.7
	[742]	p =	22	68	26	3	62	3	67	72

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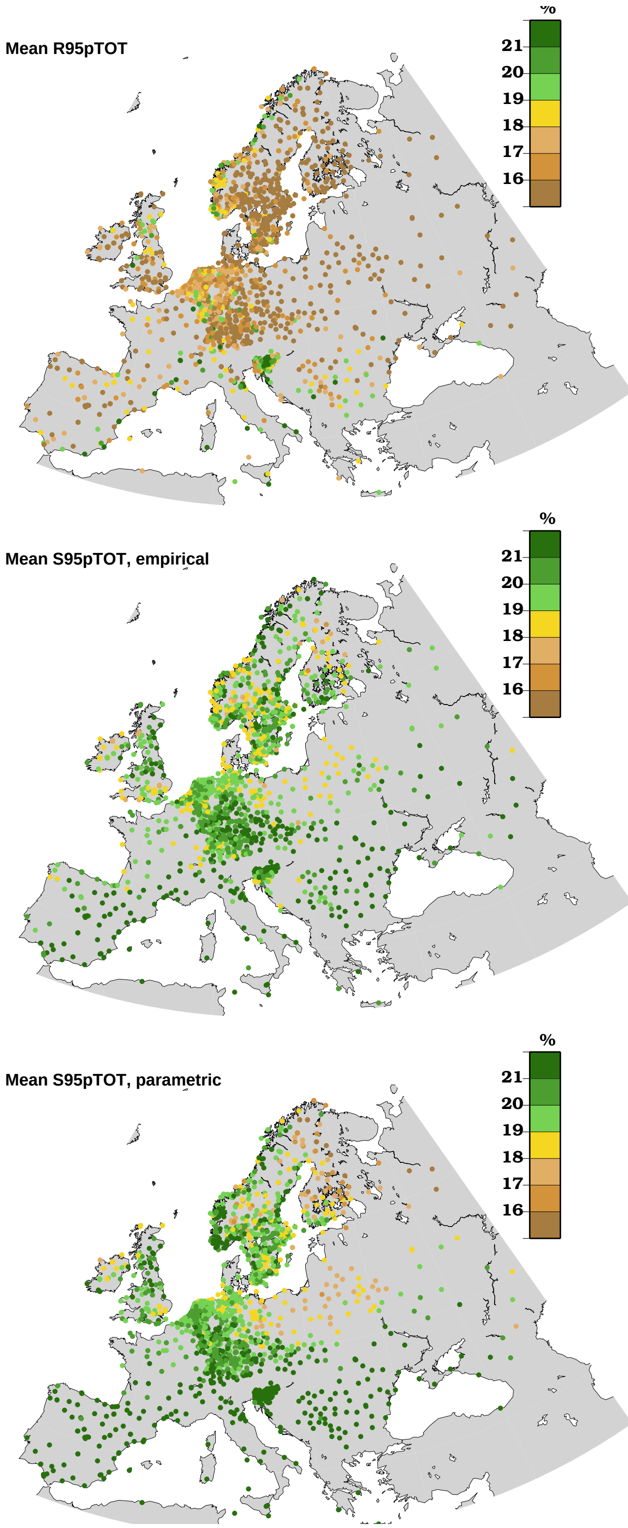

FIG. 1. Climatological (1961-2010) mean of R95pTOT (top) and of S95pTOT empirical (middle) and parametric (bottom) for the DJF season.

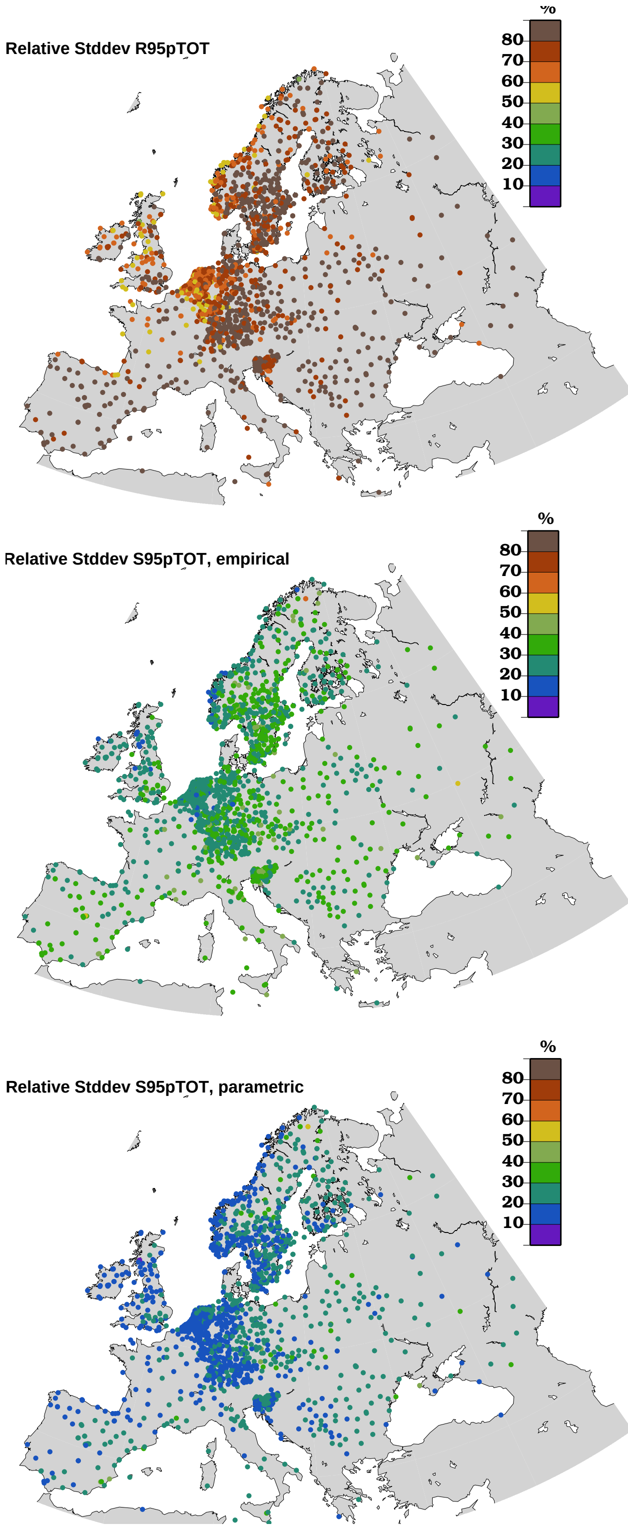

FIG. 2. Relative climatological (1961-2010) standard deviation (with respect to the climatological mean) of R95pTOT (top) and of S95pTOT empirical (middle) and parametic (bottom) for the DJF season.

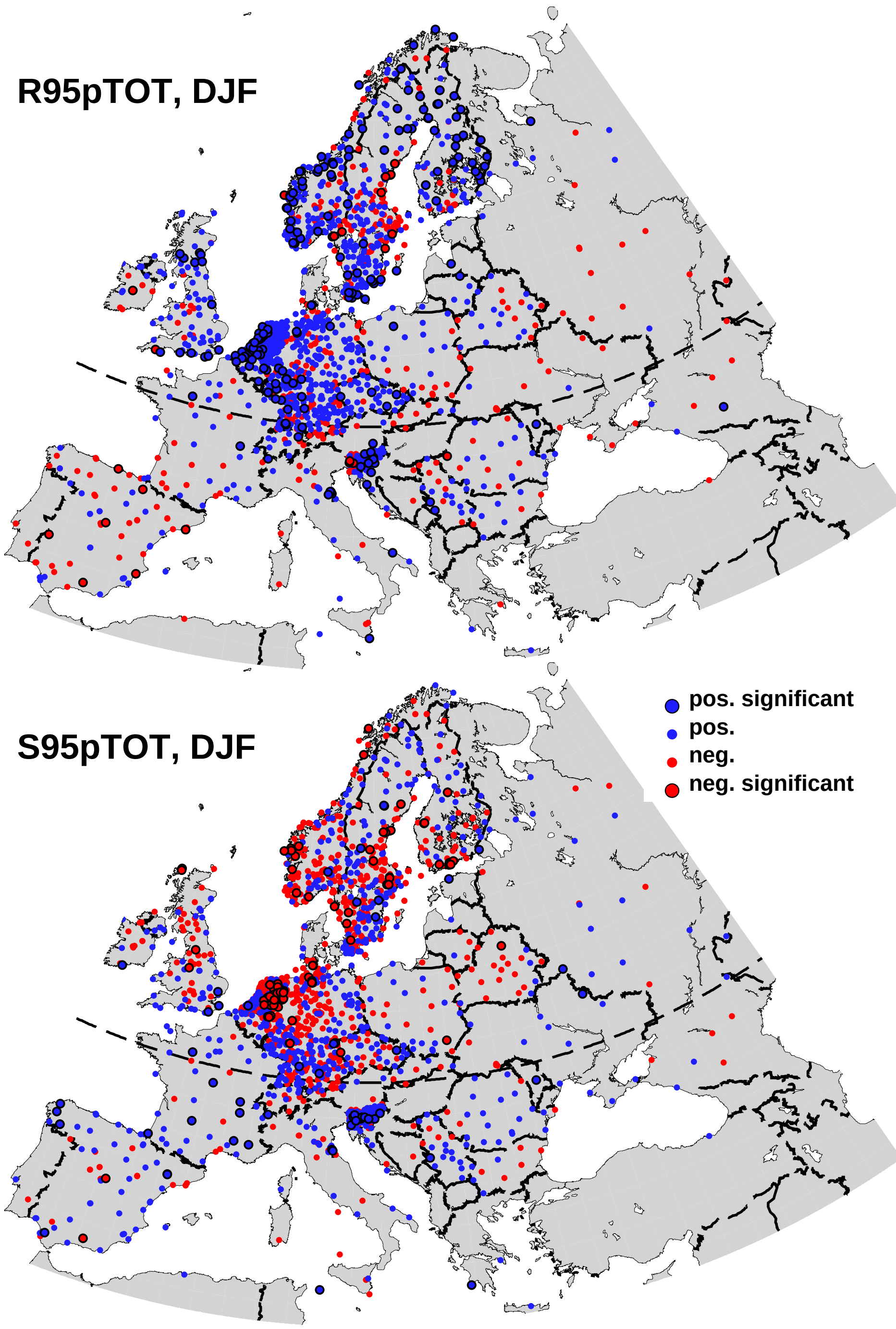

FIG. 3. Sign and significance of the trends in R95pTOT (top) and S95pTOT (bottom) for the DJF season. Decreases (negative MK values) are shown in red, increases in blue. The positions encircled in black are associated with significance at the 5%-level (two-sided test). The dashed curve marks the 48th parallel.

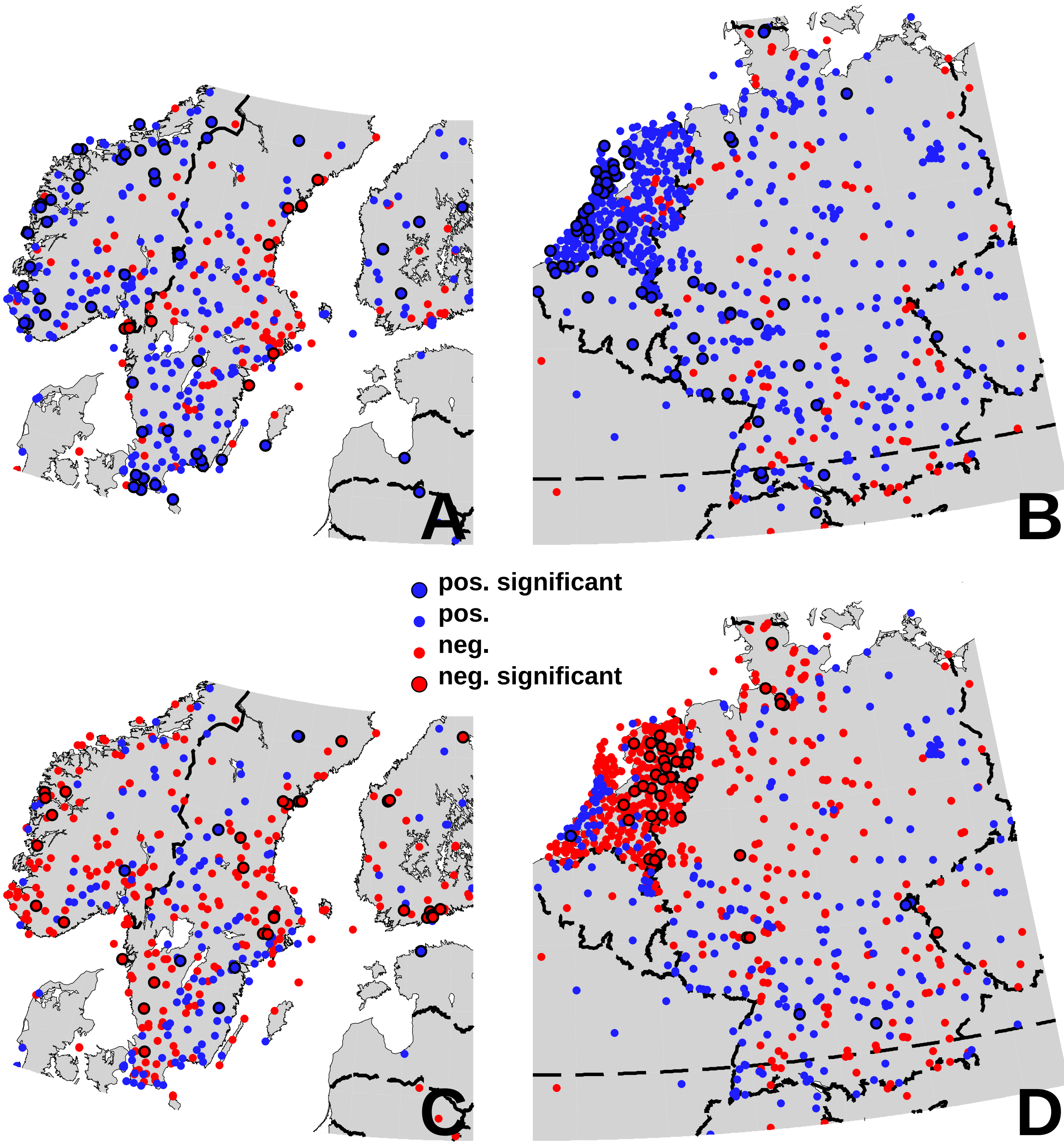

FIG. 4. Sign and significance of the trends in R95pTOT (panels A,B) and S95pTOT (panels C,D) for the DJF season, zoomed in on the regions 'NorSwe' (panels A,C) and 'NetGer' (panels B,D).

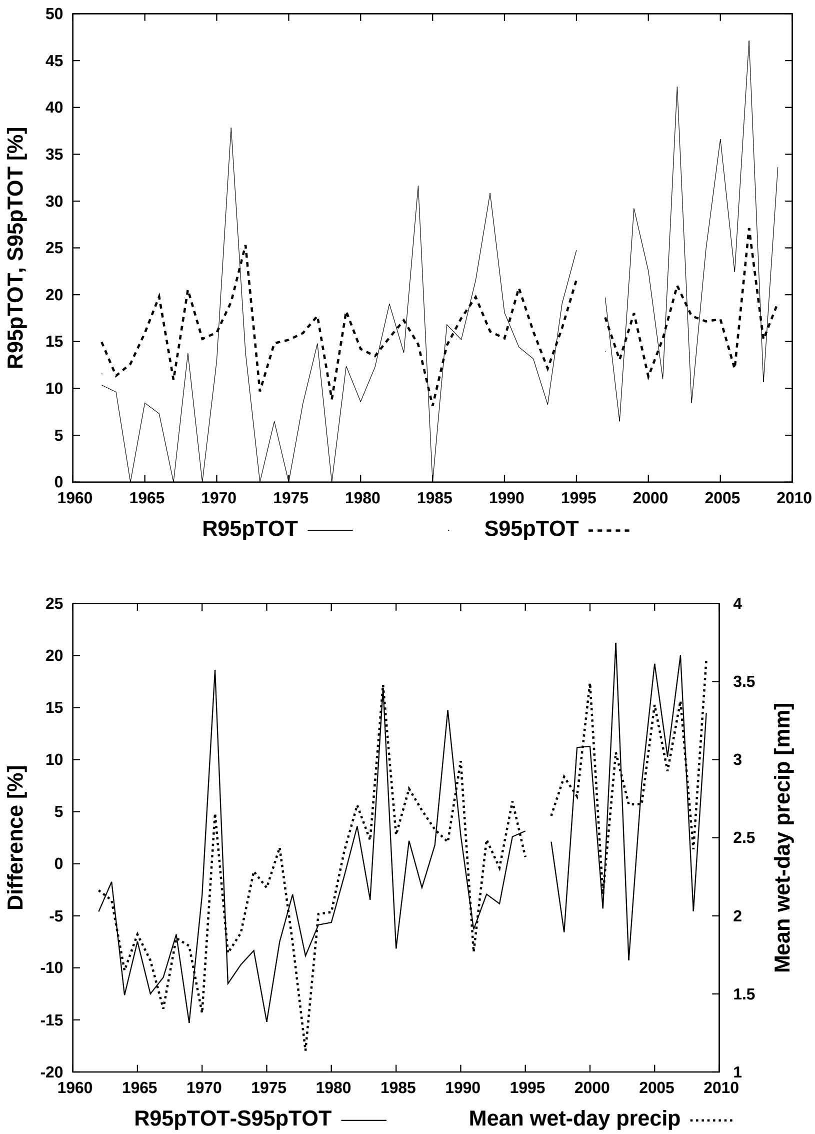

FIG. 5. Comparison between the seasonal series (for the DJF season) of R95pTOT and S95pTOT for Enonkoski Simanala (Finland). The upper panel shows the indices themselves. The lower panel compares the differences between R95pTOT and S95pTOT with the mean wet-day precipitation μ_w (different ordinates are used).

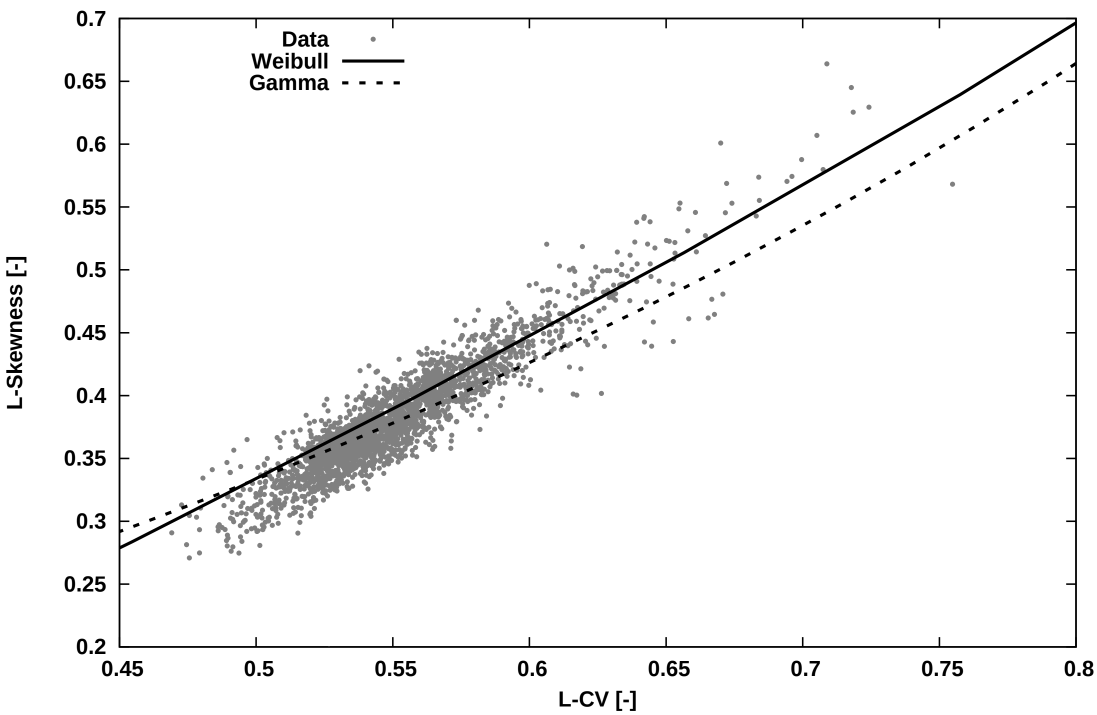

Fig. 6. Sample L-Skewness versus sample L-CV for the wet-day precipitation amounts (reduced by the wet-day threshold) in each separate DJF season and for every station. The theoretical curves representing the Weibull (solid black line) and Gamma distribution (dashed black line) are also shown.

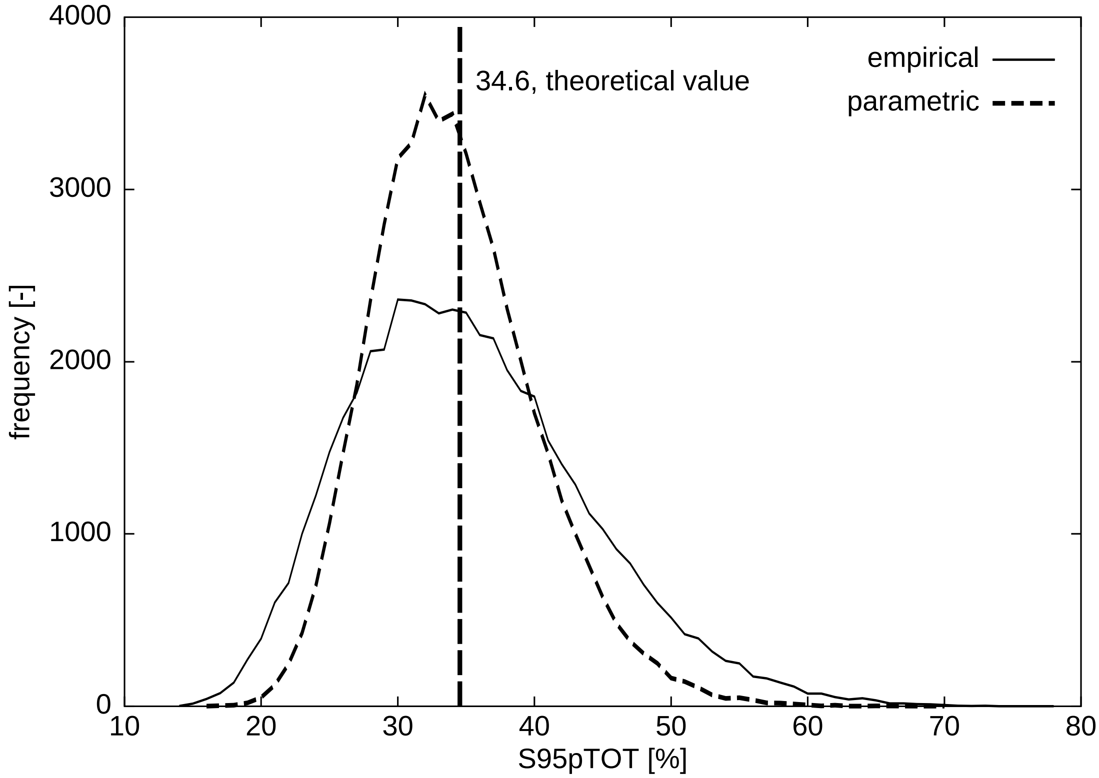

FIG. 7. Histogram of S95pTOT estimated from sets of 50 000 simulations from a Weibull distribution with $c = 0.6$ and $\alpha = 1.0$. In each simulation S95pTOT was estimated empirically as well as by means of a fitted Weibull distribution ('parametric'). The simulation length n was varied according to a binomial distribution with $p = 0.6$ and $N = 100$, to simulate the effect of a varying number of wet days within a season. The dashed vertical line marks the theoretical value of 34.6% (based on the chosen shape parameter).